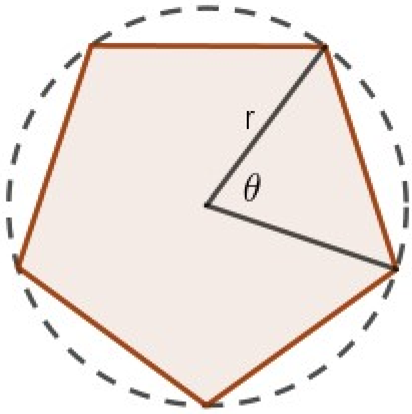
Platonic Solids

Derive the surface area of the regular polyhedrons as a function of the radius of a circle that circumscribes a polygonal face.



For a regular n-gon the angle, θ (see figure above) is simply $\frac{2\,\pi}{n}$ where n is the number of sides. The other two angles in the triangle shown are each $\frac{\pi-\theta}{2}$ or $\pi\left(\frac{1}{2}-\frac{1}{n}\right)$

We can use the sine rule to find the length, Ln, of a side of the polygon.

$$\frac{L_n}{sin\left(\frac{2\,\pi}{n}\right)}=\frac{r}{sin\left(\pi\left(\frac{1}{2}-\frac{1}{n}\right)\right)}$$

This simplifies to: $$L_n=2\cdot r\cdot sin\left(\frac{\pi}{n}\right)$$

The area of the triangle is: $$A_T=\frac{1}{2}\cdot L_n\cdot r\cdot cos\left(\frac{\theta}{2}\right)$$

or: $$A_T=r^2\cdot sin\left(\frac{\pi}{n}\right)\cdot cos\left(\frac{\pi}{n}\right)$$

or: $$A_T=\frac{1}{2}\cdot r^2\cdot sin\left(\frac{2\,\pi}{n}\right)$$

The area of the n-sided polygon is therefore: $$A_n=\frac{n}{2}\cdot r^2\cdot \sin\left(\frac{2\,\pi}{n}\right)$$

as there are n triangles in the polygon.

There are five Platonic solids:

Tetrahedron:	4 faces, 6 edges, 4 vertices.	Triangular faces	(n=3).
Cube:	6 faces, 12 edges, 8 vertices.	Square faces	(n=4).
Octahedron:	8 faces, 12 edges, 6 vertices.	Triangular faces	(n=3).
Dodecahedron:	12 faces, 30 edges, 12 vertices.	Pentagonal faces	(n=5).
Icosahedron:	20 faces, 30 edges, 12 vertices.	Triangular faces	(n=3).

The total surface area of the Platonic solids can therefore be expressed as:

$$A(f,n,r) := f \cdot \frac{n}{2} \cdot r^2 \cdot \sin\left(\frac{2\,\pi}{n}\right)$$

where f is the number of faces.

Create a calculator where the user can select a regular polyhedron by its name or number of sides, and for a given radius of the circumscribed circle of the polygonal face, the surface area will be calculated.

Define function to select polyhedron parameters given initial letter of polyhedron name:

$$PVals(P) := \begin{bmatrix}4\\3\\6\\4\end{bmatrix} \cdot (P = \text{“T”}) + \begin{bmatrix}6\\4\\12\\8\end{bmatrix} \cdot (P = \text{“C”}) + \begin{bmatrix}8\\3\\12\\6\end{bmatrix} \cdot (P = \text{“O”}) + \begin{bmatrix}12\\5\\30\\12\end{bmatrix} \cdot (P = \text{“D”}) + \begin{bmatrix}20\\3\\30\\12\end{bmatrix} \cdot (P = \text{“I”})$$

Now define polyhedron parameters calculator function:

$$PolyhedronParameters(p,r) := \begin{bmatrix}\text{“Area”} & \text{“Faces”} & \text{“Edges”} & \text{“Vertices”}\\ A\left(PVals(p)_0, PVals(p)_1, r\right) & PVals(p)_0 & PVals(p)_2 & PVals(p)_3\end{bmatrix}$$

Example calculation:

Select the initial letter of the polyhedron of interest (example is "T")
and the desired radius (example is 1):

$$p := \text{“T”} \qquad r := 1$$

$$PolyhedronParameters(p,r) = \begin{bmatrix}\text{“Area”} & \text{“Faces”} & \text{“Edges”} & \text{“Vertices”}\\ 5.196 & 4 & 6 & 4\end{bmatrix}$$

The Euler characteristic, V-E+F, is 2 for all the polyhedra.

Graph the surface area as a function of the number of faces.

$i := 0..4$

$r := 100$

$P := \begin{bmatrix} \text{"T"} \\ \text{"C"} \\ \text{"O"} \\ \text{"D"} \\ \text{"I"} \end{bmatrix}$

$\begin{bmatrix} Area_i \\ Faces_i \end{bmatrix} := \begin{bmatrix} PolyhedronParameters\left(P_i, r\right)_{1,0} \\ PolyhedronParameters\left(P_i, r\right)_{1,1} \end{bmatrix}$